HOUGHTON MIFFLIN HARCOURT

February 15, 2011

Mr. Jeff Zaring
State Board of Education Administrator
Indiana Department of Education
Room 225 State House
Indianapolis, IN 46204

Attention: Mr. Jeff Zaring, Administrator

Dear Dr. Bennett and Members of the State Board of Education,

We respectfully request that the State Board of Education reconsider the assessment of the Kindergarten – Grade 2 levels of Houghton Mifflin Harcourt's elementary mathematics program *Math Expressions*. *Math Expressions*, Kindergarten – Grade 2, was listed as "Unsatisfactory" after review by the Dana Center and Indiana teachers despite conflicting recommendations by the two groups. It is our opinion the reviews by both groups were not thorough and were subjective, and therefore led to inconsistencies and contradictions between the evaluation of individual standards and overall ratings.

To begin, reviewers neglected to review both the Teacher Editions and Student Activity Books, which was the intent of the publisher. Both Teacher Editions and Student Activity Books were provided in our state submission package. Both Teacher Editions and Student Activity Books were referenced in the *Math Expressions*/Common Core State Standards correlation provided in our state submission package. The *Math Expressions* Teacher Editions contain rich questioning and Math Talk prompts that help develop the depth of understanding required by the Common Core State Standards. It is clear from the written comments by the reviewers that only one component was examined as part of the evaluation. By not reviewing both of these important components the committee missed key elements of our Common Core State Standards coverage.

That being said, for *Math Expressions*, the reviewers assigned a rating score of 3 or 4 (strong rating) for 68% of the criteria for Kindergarten, 83% of the criteria for Grade 1 and 65% of the criteria for Grade 2. We believe if both the Teacher Editions and Student Activity Books were evaluated these percentages would be significantly higher. Attached here are responses and citations for those content standards where the reviewer assigned a rating score of 1 or 2 (weak rating). We believe this document demonstrates that when both Teacher Edition and Student Edition are evaluated it is clear Math Expressions, Kindergarten – Grade 2, completely aligns to the Common Core State Standards.

In regard to the Standards for Mathematical Practice, once again, the Dana Center did not review both the Teacher Editions and Student Activity Books as part of their evaluation for all standards. For a true understanding of how we integrate and provide complete coverage of these standards, please refer to the attached list of comprehensive responses and citations for all Mathematical Practices for *Math Expressions,* Kindergarten – Grade 2.

In closing, Dr. Karen Fuson, the author of Math Expressions, served as a member of the Feedback Team for the Common Core State Standards. The program reflects both the intent and the specifics of the Common Core State Standards. And, reviewers rated Math Expressions grades 3-5, as Satisfactory

Thank you for reconsidering these Houghton Mifflin Harcourt instructional materials for adoption by the teachers of Indiana.

Sincerely,

John Sipe

John Sipe
Senior Vice President, National Sales Manager
Houghton Mifflin Harcourt

Math Expression Grade 2 Response

Common Core Standard
2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

In *Math Expressions*, children use manipulatives such as counters and connecting cubes to group objects to determine if they are odd or even. Then this is done symbolically with drawings and numbers and related to an equation to express the even numbers as doubles (equal addends). Alternate teaching approaches are available in the side column of the Teacher Edition. Determining whether a number is even or odd is reinforced in the Home or School Activity, Differentiated Instruction Activities, and in Homework and Remembering.

Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s:
In *Math Expressions* Teacher Edition Volume 1, pages 356–357, children make pairs with connecting cubes to determine if sets of cubes have an odd or even number. This is then reinforced on Student Book page 157 where children pair circles to determine if the group of circles has an odd or even number.

Write an equation to express an even number as a sum of two equal addends:
In *Math Expressions* Teacher Edition Volume 1, pages 260–261, children use circles separated into two groups to decide if the group of circles has an odd or even number. Children count the circles writing numbers inside, alternately between the two groups. Then children relate the even numbers to an equation of doubles (two equal addends) such as $3 + 3 = 6$.

Supporting citations for Standard 2.OA.3
It is clear from the following citations that children continually determine whether a group of objects has an odd or even number of members:

Teacher Edition Volume 1
- Page 261 **Alternate Approach** (An alternate approach of sorting counters into equal groups or pairing them is presented)
- Page 265 **On Level Differentiated Instruction** (Determine if a number is odd or even)
- Page 266 **Literature Connection** *Even Steven and Odd Todd* (Sort the numbers in the story into a list of odd and even numbers)
- Page 356-357 (group connecting cubes)
- Page 359 **Intervention Differentiated Instruction** (Pair connecting cubes to decide if group of cubes has an odd or even number of members)

Student Book Volume 1
- Page 157 (pair circles)

Common Core Standard
2.NBT.1b The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

In *Math Expressions* Teacher Edition Volume 1, pages 310–314, children begin by understanding that 100 means 1 hundred, 0 tens and 0 ones using secret code cards as they count beyond 100 by tens and ones. Children draw a square around 10 tens to represent a quick hundred which they will now use to represent hundreds with a drawing.

Math Expressions also presents the meaning of the numbers 100 through 900 using dollar bills with 100 pennies on the back to count by hundreds to 1,000. They observe that there are no tens and ones only bills (hundreds) that represent one or more hundreds. The number is also recorded on the board to show 0 in tens place and 0 in ones place as they count the bills. The bills with the pennies on the back can be found on Student Book page 339. Accompanying support is found on Teacher Edition pages 756-757. Children then write word names for 100– 900 using a chart that shows 100–900 in words as well as numbers to reinforce the meaning of the symbols and the meaning of the numbers without tens or ones. This is found on Student Book page 346.

Supporting citations for Standard 2.NBT.1b
It is clear from the following citations that children understand the meaning of the numbers 100, 200, 300, 400, 500, 600, 700, 800, and 900 as a number of hundreds and 0 tens and 0 ones:

Teacher Edition Volume 2
- Pages 765- 767 (Children represent 3-digit numbers in various ways: secret code cards, quick hundreds, sticks and circles and see that the numbers 100–900 have only quick hundreds no sticks or circles. Children then do the reverse, write numbers given secret codes cards or a quick picture.)
- Page 774 (provides teaching support for dollar bills representing hundreds on Student Book page 346)

Student Book Volume 2
- Page 339 (hundreds represented by dollar bills)
- Page 346 (write word names for numbers)

Common Core Standard
2.NBT.2 Count within 1000; skip-count by 5s, 10s, and 100s.

In *Math Expressions*, children use quick pictures, gesturing, and patterns to count within 1000. Skip counting by 5s and 10s is done by writing number patterns and using fingers and hands. Skip counting by 100s is done using manipulatives (dollar bills) they cut out from a Student Book page with 100 pennies on the back of the bills. Counting within 1000 and skip counting by 5s, 10s, and 100s is reinforced in the Quick Practice, Home or School Activities, Differentiated Instruction Activities, and in Homework and Targeted Practice.

Count within 1000:
Math Expressions begins counting within 1000 on Student Book page 182 where they count from 1 to 100. Teaching support is found on Teacher Edition page 419 where children look for patterns in the 100s interval which will help them in the rest of the hundreds intervals to 1000. Counting within 1000 continues with emphasis on counting over hundreds by drawing quick pictures to represent the numbers to visualize what happens when a new hundred is reached. An example can be found on Teacher Edition pages 758–759. An example of gesturing to count numbers is found on Teacher Edition page 760. (Students count to ten with their fingers, flash 10 fingers to represent 10s, and draw a box in the air to indicate a new hundred.)

Skip-count by 5s, 10s, and 100s:
In *Math Expressions* children use skip counting by 5s and 10s to find patterns in numbers and as a readiness skill for multiplication by 5s. An example can be found on Student Book page 481 where students skip count by 5 and 10s starting at numbers such as 3, 100,and 119. The teaching support on Teacher Edition page 1075 includes looking for patterns in numbers. Children skip count by 100s using the pennies sides of the dollar bills on Student Book page 339 with teaching support on Teacher Edition page 756.

Supporting citations for Standard 2.NBT.2
It is clear from the following citations that children continually count within 1,000 and skip count by 5s, 10s, and 100s:

Student Book Volumes 1 and 2
- Page 182 (count to 100)
- Page 339 (bills with pennies)
- Page 345 (count by ones and 10s from 396 to 930)
- Page 431 (count by 5s from 1-50)
- Page 433 (count rows of 5 to 40 and relate count by 5s to multiply by 5)
- Page 481 9skip count by 5s and 10s starting at 3, 100, and 119. Count by 10 from 8, 10, 92, and look for patterns in numbers)
- Page 482 (use a calculator to count backward and forward by 10.)

Teacher Edition Volumes 1 and 2
- Page 422 **Art Connection** (create a pattern)
- Pages 772–773 (quick pictures and gesturing activities)

- Page 756 (support for Student page 339 count by 100s)
- Page 775 **Differentiated Instruction Activities** (counting within 1000)
- Page 776 **Technology Connection** (skip count using calculator)
- Page 953 **Quick Practice** (skip counting by 5s to prepare for multiplication by 5s.)
- Page 974 (support for Student page 431 count by 5s and 10s)
- Page 980 (support for Student page 433 count by 5s and relate to multiplication by 5.)
- Pages 1075–1176 (support for Student pages 481–482)
- Page 1077 **Differentiated Instruction Activities** (skip count by 5s and 10s both forward and backward)
- Page 1078 **Real World Connection (**counting things that come in 5s or 10s (nickels, starfish arms, toes on a person, stickers on a page)

Homework and Targeted Practice Example

- Pages 122–123

Common Core Standard
2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

In *Math Expressions* children are involved in meaningful activities that will help them recognize the meaning of hundreds and further their understanding of place value. They will represent 3-digit numbers using quick drawings and secret code cards in expanded form to prepare for comparing two 3-digit numbers based on the meanings of the hundreds, tens, and ones digits. Examples can be found on Teacher Edition pages 756–760 and 765–767.

Math Expressions also reinforces the meaning of the symbols >, + and < to record the results of comparison. They compare numbers using these symbols. This can be found on Teacher Edition pages 144–145.

Supporting citations for Standard 2.NBT.4
It is clear from the following citations that children continually reinforce the meaning of the hundreds, tens and ones.

Teacher Edition Volume 2

- Pages 756–760 (quick pictures)
- Page 761 **Challenge Differentiated Instruction Activity** (Children order 3-digit numbers using quick pictures)
- Page 765-767 (Children show 3-digit numbers in various ways: secret code cards, quick hundreds, sticks and circles. Children then do the reverse, write numbers given secret codes cards or a quick picture)
- Page 813 **Challenge Differentiated Instruction Activity** (Compare 3-digit sums)

Common Core Standard
2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.

In *Math Expressions*, children are introduced to three methods for adding four two-digit numbers: (1)Adding a number to the on going total after adding the first two addends. (2) Add two 2-digit numbers, then add the other two 2-digit numbers, and last add the totals, making proof drawing for all addends to show the combinations of tens and ones. (3) Add the four two-digit numbers in a column with proof drawings so children see they may get more than one new ten. This can be found on Teacher Edition pages 398-399. Children were also introduced to the New Groups Below Method and the Expanded method for adding two two-digit numbers on Student page 177 with accompanying support on Teacher Edition page 406 to prepare children for adding four two-digit numbers.

Math Expressions presents the properties of 0 and 1 for addition on Teacher Edition pages 14–15 through solving story problems. The Commutative Property is presented as "Switching Addends" on Teacher Edition page 41. The Associative Property is introduced on Teacher Edition page 150 with alternate teaching approach.

Supporting citations for Standard 2.NBT.6
It is clear from the following citations, any pages with word problems, and Homework and Remembering that children add up to four two-digit numbers:

Teacher Edition Volume 1
- Pages 14-15, 41, 150 (properties of 0 and 1 for addition, commutative and associative properties)
- Page 350 (100 plus a number)
- Pages 398-399 (3 methods for adding four two-digit numbers)
- Page 406 (New Groups Below and Expanded methods)
- Page 420 (practice adding two-digit numbers)

Student Book Volume 1
- Page 173 (word problems presented for 3 methods of adding)
- Page 177 (add two-digit money amounts)
- Page 183 (use code to get numbers, then add)

Remembering Sample page 116

Common Core Standard
2.NBT.8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

Math Expressions presents mentally adding 10 to a given number and mentally subtracting 10 from a given number through skip counting forward and backward starting at a numbers other than 0. This can be found on Student Book pages 481-482 and Teacher Edition pages 1074-1076. Student Book page 349 can be used to check students proficiency with adding 10s and 100s mentally. Using mental math to add and subtract is also reinforced in the Quick Practice before each lesson. An example of a Quick Practice that does this can be found on page 403 of the Teacher Edition.

In *Math Expressions*, children use mental math to add 10 whenever they add two-digit numbers with a one in the tens place. They use mental math to add 100 whenever they add 3-digit numbers with a 1 in the hundreds place. Children use mental math to subtract 10 whenever they subtract a two digit number with a 1 in the tens place of the subtrahend. They use mental math to subtract 100 whenever they subtract a three digit number with a 1 in the hundreds place of the subtrahend.

Supporting citations for Standard 2.NBT.8
It is clear from the following citations and any time they add or subtract two- or three-digit numbers that children mentally add 10 or 100 to a given number or mentally subtract 10 or 100 from a given number.

Teacher Edition Volumes 1 and 2
- Page 403 **Quick Practice**
- Page 756 (count by 100s)
- Page 772-773 (count by tens orally and with gesturing)
- Page 775 **On Level Differentiated Instruction** (find 10 less)
- Page 784 (teaching support for Student page 349)
- Pages 1074–1076 (skip counting forward and backward by 10)
- Page 1077 **Differentiated Instruction Activities** (skip count by 10s both forward and backward)

Student Book Volume 2
- Page 339 (count by 100s)
- Page 345 (count by 10s)
- Page 349 (mentally add 10 and 100)
- Page 437 (Exercise 1, add 10 mentally)

Common Core Standard
2.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters.

In *Math Expressions* children estimate the length of their hand, arm, pencil, window, door and their heights in the appropriate unit, centimeters or meters. They also discuss personal referents for these units of measure. This can be found on Student book pages 357 – 358 with teaching support on Teacher Edition pages 915–916.

Math Expressions also covers the customary units of measure, inches and feet. Children estimate the measure of their hands, classroom objects, and line segments. A personal referent for an inch is also discussed. This can be found on Student Book pages 470 and 473 with teaching support on Teacher Edition pages1049 and 1051.

Supporting citations for Standard 2.MD.3
It is clear from the following citations that children estimate lengths using units of inches, feet, centimeters, and meters:

Teacher Edition Volume 2
- Pages 915- 916 (estimate with centimeter and meter)
- Page 923 **Differentiated Instruction** (estimate centimeter and meter)
- Pages 1049 and 1051 (estimate with in. and feet)

Student Book Volume 2
- Pages 357–358 (estimate with centimeter and meter)
- Pages 470 and 473 (estimate with inches and feet)

Common Core Standard
2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

Word problem form is an essential part of the *Math Expressions* curriculum as children connect the outside world with the classroom. There are many mathematical situations that require children to understand the relationship between known and unknown quantities. Children are frequently asked to create their own story problems and discuss the language as well as solve the problem for the unknown. A box is used at this level to represent the unknown number. An example of writing a word problem using a drawing for perimeter (in same units) can be found at the bottom of Teacher Edition page 399. Student page 173 gives children an opportunity to use drawings with lengths labeled to write an equation and solve for the unknown with teaching support on Teacher Edition page 398.

Supporting citations for Standard 2.MD.5
It is clear from the following citations that children use addition and subtraction within 100 to solve word problems involving lengths and equations with a symbol for the unknown number to represent the problem

Teacher Edition Volumes 1 and 2
- Page 142 **Social Studies Connection Activity** (use drawing of a map to write and solve story problems)
- Page 183 **Differentiated Instruction Activity** (rectangles with lengths for finding perimeter)
- Pages 398-399 (use drawing with same unit lengths to solve a problem)
- Page 680 **Social Studies Connection Activity** (make a map and find the distances between places)
- Page 884 **Social Studies Connection Activity** (write and solve a word problem about average depths of Great Lakes)
- Page 916 (write their own questions about height data collected)
- Page 1053 **Challenge Differentiated Instruction Activity** (write and solve equations about distances)

Student Book Volume 1
- Page 173 (use drawings with lengths to write and solve equations to solve word problems)

Common Core Standard
2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

In *Math Expressions* students partition rectangles into rows and columns of the same size squares using squares made of cardstock and count them to find the area of a figure. Examples can be found on Student Book pages 333–334 with teacher support on Teacher Edition pages 746-747.

Supporting citations for Standard 2.G.2
It is clear from the following citations that children partition a rectangle in rows and columns of same-size squares and count to find the total number of them:

Teacher Edition Volume 2

- Pages 746–747 (partition with cardstock or use shaded part of grid to count and find area)
- Page 749 **Differentiated Instruction Activities** (square tiles, pattern blocks, and grids are used to partition)
- Page 750 **Science Connection** (trace leaves on grid paper and count squares to find area)

Student Book Volume 2

- Pages 333–334

Mathematical Practice 1: Make sense of problems and persevere in solving them.

Mathematical Practice 1 begins by stating: Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. In the only explicit statement about this standard for younger children, it states: Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. These statements are the very essence and core of the *Math Expressions'* algebraic problem-solving approach. This meaning-making approach begins in Kindergarten and focuses on the Common Core OA problem types. This approach supports students to **understand the situation, represent the situation, decontextualize (MP2) to find a solution**, and **contextualize (MP2) to see that the answer makes sense.**

Understand the Situation

In early units, Kindergarten through Grade Two students act out, solve, and pose addition and subtraction world problems using Scenarios that come from family life experiences, pictures, or real-life scenes. Teachers encourage students to visualize the addition or subtraction story problem and even retell the story in their own words, all of which allows students to analyze the information, make meaning of the problem, and focus on *understanding* the situation and *understanding* the meaning of the question being asked.

For example, Kindergarten students in Unit 2 Lessons 3 and 5 act out and discuss real-life family situations. Teachers encourage students to retell the stories in their own words and help focus student attention to the addition and subtraction situations, all of which help students build confidence in their explanations and develop a competent math vocabulary. In Unit 3 Lesson 1, students visualize a park scene where children are playing with balls. *There is 1 child playing with balls. Imagine more children come to play. What if 5 more children come? How many children will be playing with balls?* First, students think about and discuss what's going on in the scene and make sense of it all. Then the teacher records their visualization with an equation for the unknown total ($1 + 5 = \square$) to represent the situation symbolically. Finally, students solve the story problem with fingers or objects (square inch tiles or cubes)- strategies they are comfortable and competent with using at this grade level. First and Second graders use these same type of scenario structures as seen in Unit 2 Lessons 8 and 11 (Grade 1) and Unit 1 Lesson 11 (Grade 2).

As students advance through first and second grade, *Math Expressions* continues to expose students to many types of word problems and encourages them to comprehend the situation (or *decontextualize* or abstract the situation) in order to solve it. Extensive research from all over the world has identified four basic classes of addition and subtraction problems. These problems can be identified as problems that involve: 1.) joining, 2.) separating, 3.) part-part-whole relations, and 4.) comparison relations.

In *Math Expressions*, students are exposed to these types of problems (known as unknown partner, unknown total, change, collection, and comparison). Once students have seen a variety of the same word problem types over time, they start to look for similar patterns and structures that helps them decipher which problem they are solving for, making the problem-solving process much more efficient and accurate.

In Grade 1, these word problem types can be seen in Unit 2 Lessons 1-3, 11-14, Unit 3 Lessons 1-12, Unit 6 Lessons 8-9, and Unit 9 Lessons 1, 6-9. In Grade 2, they can be seen in Unit 3 Lessons 1- 3, 5-7, Unit 9 Lessons 15-16, and Unit 11 Lessons 13, 20, 21.

Represent and Solve

Once students have found meaning in the problem, they share ideas of how they may solve the problem and then use various methods to solve it (i.e. acting it out, solving with fingers and/or other concrete objects, making math drawings/models, writing expressions (K) and/or equations (Grades 1-2). At this stage of development, writing the whole equation might be difficult for students in Kindergarten, so teachers will record the full equation to relate the visual representation with the numerical/symbolic one.

Check that Answers Make Sense

In order to check for accuracy and explain their thinking to others, students use the various representations discussed above (i.e. models/ drawings, concrete objects, symbols/equations) to help explain and justify their work. This frequent exchange of mathematical ideas and problem-solving strategies is known as Math Talk, a key built-in component of *Math Expressions.* On a daily basis, students are encouraged to communicate their ideas (using precise mathematical language), explain their thinking, and critique the reasoning of others. This is done through a Solve, Explain, Question, and Justify classroom structure in which students make math drawings at the board along with their numerical solution method. Then, two or three students explain their methods while other students ask questions or critique their reasoning in order to stimulate more complete and adapted explanations. The teacher facilitates from the side or the back of the room to increase the amount of direct student-to-student dialogue. Initially, for any new math topic, the teacher may also need to model full explanations of some methods and help students explain more fully. Each Teacher's Guide also contains sample questions, explanations, and student-teacher dialog (Math Talk in Action) to help teachers build a more advanced Math Talk classroom.

See also Mathematical Practices 2, 4, and 8 with respect to representing and solving problems and Mathematical Practices 3 and 7 for elaboration on Math Talk.

Supporting citations for Mathematical Practice 1:

It is clear from the following citations that children:

- analyze and make conjectures about how to solve a problem.
- plan, monitor, and check their solutions.
- determine the reasonableness of their answers and justify their reasoning.

Kindergarten- Teacher Edition: 138-139, 243, 260, 408-409, 436, 464, 466

Unit 1 establishes the basic foundation of counting objects and seeing groups of objects in the real world. Now in Unit 2, students are ready to work with those objects by acting out and solving and posing addition and subtraction problems through the use of scenarios. As seen on pages 138 and 139, the teacher elicits a family-meal story from a child and then helps the students focus on the mathematical aspects of the situation. Students are encouraged to visualize the situation (as also seen on page 243) and even ask clarifying questions to help them make meaning of the problem and understand the question being asked (How many plates are on the table?). Then, a group of students use real objects (paper plates, chairs, and tables) to act out the solution method while other students use their fingers to help them solve the problem. As students progress throughout the year, visual representations and/or math drawings (pages 408 and 409) can be used to solve the problem, check their work, and help justify their thinking to the class (as seen on page 466).

Grade 1- Teacher Edition: 122, 128-129, 134-135, 161, 178–180, 184–185, 190–191, 196–197, 224–225, 231–232, 248–250, 260; Student Activity Book: 67, 85, 87, 97

First graders continue to use scenarios in order to relate to and solve word problems (as seen on pages 161 and 178). Students are asked to "think about what's happening in the scene" and to focus on what the vocabulary word "minus" means in the context of the story (page 179). Initially students plan to solve the problem by acting out the situation with real paper plates or role playing the actions, but quickly, they transition to use concrete (then abstract drawings) to represent and solve the problem (pages 178-179). As students begin to see a variety of word problem types, they are then able to recognize the situation, plan accordingly, represent the situation with an equation and then use a "proof drawing" to check their answer for accuracy before justifying their thinking to the class (page 184). As students continue to become more confident with their problem solving approaches and their mathematical explanations become more advanced, they even begin to understand and explain why their answers are reasonable (pages 197 and 249).

Grade 2- Teacher Edition: 76–77, 196–198, 202–204, 208–210, 222–224, 235; Student Activity Book: 89, 93, 95,101 107

Second graders also begin visualizing word problems with the use of scenarios in a yard sale scene (pages 76–77). To help students make meaning of the situation, teachers elicit prior knowledge of buying and selling items in order for students to understand that addition and subtraction will be the mathematical operations in use. To represent the subtraction situation (10¢ – 9¢ = 1¢) students act out the purchase with concrete coin strips. By Unit 3, however, students begin to see a variety of word problem types (building on from the ones in Grade 1) and over time are able to recognize the situation, plan accordingly, represent the situation with an equation and then use a "proof drawing" to check their answer for accuracy before justifying their thinking to the class using the Solve and Discuss structure (page 208, 228).

See also the Houghton Mifflin Harcourt Math Expressions © 2009 correlations [Grades K, 1, 2] to the Common Core State Standards Initiative for Mathematics (2010) for additional opportunities to see the integration of the math practices.

Mathematical Practice 2: Reason abstractly and quantitatively.

The Common Core Initiative believes mathematically proficient students create a coherent representation of the problem at hand, consider the units involved, attend to the meaning of quantities, not just how to compute them, and flexibly use different properties of operations and objects to solve problems. *Math Expressions* students reason abstractly and quantitatively as they decontextualize (mathematize) a problem situation to create a coherent representation of the problem with objects, math drawings, and equations. They label their answers with the appropriate unit. They represent tens and ones (later, hundreds, tens, and ones) with drawings and relate these drawings to their computation, thus attending to the meaning of quantities. They informally use the commutative and associate properties to find more efficient solution methods.

Make Sense of the Problem
Story problems provide important opportunities for young learners to make sense of mathematics around them. Students often use strategies including *acting out* the problem to make sense of a solution path. Kindergarten through Grade Two students act out, solve, and pose addition and subtraction world problems using Scenarios that come from family life experiences, pictures, or real-life scenes. Teachers encourage students to visualize the addition or subtraction story problem and even retell the story in their own words, all of which allows students to analyze the information, make meaning of the problem, and focus on *understanding* the situation and *understanding* the meaning of the question being asked.

For example, Kindergarten students in Unit 2 Lessons 3 and 5 act out and discuss real-life family situations. Teachers encourage students to retell the stories in their own words and help focus student attention to the addition and subtraction situations, all of which help students build confidence in their explanations and develop a competent math vocabulary. In Unit 3 Lesson 1, students visualize a park scene where children are playing with balls. *There is 1 child playing with balls. Imagine more children come to play. What if 5 more children come? How many children will be playing with balls?* Students think about and discuss what's going on in the scene allowing the class to make sense of it all.

Consider the following second grade problem from Unit 2 Lesson 11. *Six people are swimming. Then 4 go home. How many are still swimming?* In an effort for all students to make sense of the problem, the teacher has 4 of the 6 children "dry off" and leave the scene. Followed up by a discussion, most students will make sense of the problem and know how to proceed to solve it. Another way students can make sense of the problem is to *not* focus on the key words but use the story to make sense of a solution process. "They know to begin with 6 and they will be taking something away."

Represent the Problem
Now, for students to solve problems in a mathematically proficient way, they can use a variety of strategies to represent the problem. Using the examples from Kindergarten and Grade 2 from above, we'll see how the students represented their situation and story problem.

In Kindergarten, the teacher records their visualization with an equation for the unknown total ($1 + 5 = \square$) to represent the situation symbolically. Then students can concretely solve the story problem with fingers or objects (square inch tiles or cubes)- strategies and materials they are comfortable and competent with using at this stage of mathematical development.

In Grade 2, students have more strategies and representations they can call upon to help them solve this problem. In this example, some students may draw concrete pictures of 6 children (represented by smiley faces) with a line drawn through 4 of them to show 4 of them leaving and only 2 swimmers left. Another student may still be using a visual representation, but this student uses a circle drawing of 6 circles with a line through 4 of them showing 2 people left swimming. To link the visual representation with a symbolic one, the teacher has students write an equation, which gives meaning to the quantities seen in the drawings ($6 - 4 = 2$).

Reasonable Answers and Communication
Regardless of which strategy students use to solve the problem, they are encouraged to check their answers for reasonableness. Being able to recognize if an answer is reasonable is vital to achieving competency in problem solving. Teachers encourage students to understand that problem solving is more than just finding the right or correct answer; it also involves the ability to find and classify information that is relevant to the problem. Once students have validated their results and are happy with their solution, they use Math Talk in conjunction with their representations to help justify their thinking and explain their work to the class.

For example, in Unit 3 Lesson 24 a Math Talk in Action feature outlines how Kindergarten students have made up this story problem. *There were 7 ducks in the park. Then 3 left. How many are there now?* Students engage in a discussion with each other debating whether the answer 10 or 4 is correct. One student explains that *4 is correct because there were only 7 ducks to start with, and since some left, the answer has to be less than 7 and 10 is greater than 7.* The teacher praises the student's work and has the whole class check the mathematical thinking by modeling the situation with square inch tiles.

In Grade 1 you can see examples of similar type dialogs on pages 135, 196, 238, and 249 and in Grade 2 on page 256. Also at Grade 2, on page 646, students are taught how they can use rounding to check that their subtraction methods are accurate and that their solution is reasonable.

Supporting citations for Mathematical Practice 2:

It is clear from the following citations that children:

- make sense of quantities and their relationships in problem situations.
- connect symbols, words, and or drawings to represent a given situation.
- check their answers for reasonableness.

Kindergarten- Teacher Edition: 138-139, 243, 260, 360, 408-409, 436, 464, 466

Unit 1 establishes the basic foundation of counting objects and seeing groups of objects in the real world. Now in Unit 2, students are ready to work with those objects by acting out and solving and posing addition and subtraction problems through the use of scenarios. As seen on pages 138 and 139, the teacher elicits a family-meal story from a child and then helps the students focus on the mathematical aspects of the situation. Students are encouraged to visualize the situation (as also seen on page 243) and even ask clarifying questions to help them make meaning of the problem and understand the question being asked (How many plates are on the table?). Then, a group of students use real objects (paper plates, chairs, and tables) to act out the solution method while other students use their fingers to help them solve the problem. As students progress throughout the year, visual representations and/or math drawings (pages 408 and 409) can be used to solve the problem, check their work for reasonableness (360), and help justify their thinking to the class (as seen on pages 360, 466).

Grade 1- Teacher Edition:122, 128-129, 134-135, 161, 178–180, 184–185, 190–191, 196–197, 224–225, 231–232, 238, 248–250, 260; Student Activity Book: 67, 85, 87, 97

First graders continue to use scenarios in order to relate to and solve word problems (as seen on pages 161, 178). Students are asked to "think about what's happening in the scene" and to focus on what the vocabulary word "minus" means in the context of the story (page 179). Initially students plan to solve the problem by acting out the situation with real paper plates or role playing the actions, but quickly, they transition to use concrete (then abstract drawings) to represent and solve the problem (pages 178-179). As students begin to see a variety of word problem types, they are then able to recognize the situation, plan accordingly, represent the situation with an equation and then use a "proof drawing" to check their answer for accuracy before justifying their thinking to the class (page 184). As students continue to become more confident with their problem solving approaches and their mathematical explanations become more advanced, they even begin to understand and explain why their answers are reasonable (pages 135, 196-197, 238, 249).

Grade 2- Teacher Edition: 76–77, 196–198, 202–204, 208–210, 222–224, 235, 256, 646; Student Activity Book: 89, 93, 95,101 107

Second graders also begin visualizing word problems with the use of scenarios in a yard sale scene (pages 76–77). To help students make meaning of the situation, teachers elicit prior knowledge of buying and selling items in order for students to understand that addition and subtraction will be the mathematical operations in use. To represent the subtraction situation (10¢ – 9¢ = 1¢) students act out the purchase with concrete coin strips. By Unit 3, however, students begin to see a variety of word problem types (building on from the ones in Grade 1) and over time are able to recognize the situation, plan accordingly, represent the situation with an equation and then use a "proof drawing" to check their answer for accuracy before justifying their thinking to the class using the Solve and Discuss structure (page 208, 228). Students at this stage continue to check answers for reasonableness using logic and reasoning while discussing problem solving situations, but as they become more sophisticated with their mathematics they also learn rounding skills to help them check answers for reasonableness as seen on page 256.

See also the Houghton Mifflin Harcourt Math Expressions © 2009 correlations [Grades K, 1, 2] to the Common Core State Standards Initiative for Mathematics (2010) for additional opportunities to see the integration of the math practices.

Mathematical Practice 3: Construct viable arguments and critique the reasoning of others.

Mathematical Practice 3 states this about elementary students and about students at all grades: Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

A key built-in component of *Math Expressions* is the frequent exchange of strategies and reasoning, or Math Talk. On a daily basis, students are encouraged to communicate their ideas (using precise mathematical vocabulary), explain their thinking, and critique the reasoning of others. *Math Expressions* develops a collaborative Math Talk culture of understanding, explaining, questioning, and helping.

Math Expressions introduces a Solve, Explain, Question, and Justify classroom structure in which students make math drawings at the board along with their numerical solution method. Then, two or three students explain their methods while other students ask questions or critique their reasoning in order to stimulate more complete and adapted explanations. The teacher facilitates from the side or the back of the room to increase the amount of direct student-to-student dialog. Initially, for any new math topic, the teacher may also need to model full explanations of some methods and help students explain more fully. Each Teacher's Guide also contains sample questions, explanations, and student-teacher dialog (Math Talk in Action feature) to help teachers build a more advanced Math Talk classroom.

There are two kinds of Solve, Explain, Question, and Justify classroom structures that are very effective in engaging all students in Math Talk.

Solve and Discuss
The teacher selects 4 to 5 students (or as many as space allows) to go to the classroom board and solve a problem, using any method they choose. Their classmates work on the same problem at their desks using their MathBoards (dry-erase boards). Then the teacher picks 2 to 3 students to explain their methods. Students at their desks are encouraged to ask questions and assist their classmates in their understanding.

Step-by- Step at the Board
This is a variation of the Solve and Discuss structure where several children go to the board to solve a problem. This time, however, a different student performs each step of the problem, describing the step before everyone does it. This approach is particularly useful as mathematical concepts become more advanced (i.e. multi-digit addition and subtraction). Is assists the least-advanced students the most with providing accessible, systematic methods.

The opportunity for all students to explain their math thinking over time is especially valuable for students learning English, as well as for native speakers advancing their verbal communication skills. Ultimately, developing understanding and verbal communication will aid all students in their future education and careers. In addition to verbal communication, the use of math drawings is central to Math Talk. Math drawings can show the quantities in a computation and relate them to a written numerical method or can show the situation in a word problem. The math drawings help everyone understand the student's math thinking. The special learning supports on the MathBoards enable students to learn meaningful drawings rapidly and then the open space on the MathBoards is used for math drawings. Board work reveals multiple methods of solving and it gives the teacher an opportunity to highlight selected methods and identify errors students may be making. Both the teacher and other students can correct these errors in a supportive, helping community.

Supporting citations for Mathematical Practice 3:

It is clear from the following citations that children:

- use stated assumptions, definitions, and previously established results in constructing arguments.
- analyze situations and can recognize and use counterexamples.
- justify their conclusions, communicate them to others, and respond to the arguments of others.
- can distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.
- can listen to the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Kindergarten- Teacher Edition: 2, 3, 8, 9, 275, 360, 408

Early on, Kindergarten students learn to talk about math using their classroom voices and discuss mathematical concepts seen in *Anno's Counting Book* (pages 2-3). Math Talk can clarify student's thinking and expose the entire class to other approaches. Students use a daily Counting Mat activity in which they have to arrange and rearrange tiles to represent numbers. The different arrangements lend themselves to good discussion topics around the embeddedness of numbers and an early understanding of addition and subtraction (pages 8-9). Students also describe, explain, discuss and defend their solution methods when problem solving (page 360). They learn to talk to the class, listen to their classmates, and critically analyze their own work and the work of their peers (pages 275, 408). Developing communication skills will be a year-long goal for these students, a goal they will carry on with them indefinitely.

Grade 1- Teacher Edition: 2-3, 14, 28-29, 140, 184-185, 190, 208

First grade students continue to learn to talk about math as they share math stories and engage with numbers, patterns, and shapes in the world around them (pages 2-3, 14). Math Talk can clarify student's thinking and expose the entire class to other approaches. Students begin breaking apart numbers, which helps them understand the embededness of numbers, and builds the foundation for addition and subtraction. In one activity, students discuss the various ways you can break apart the number 4, using visual representations and numerical expressions to explain their thinking (pages 28-29). Students describe, explain, discuss and defend their solution methods when problem solving (pages 184-185). They learn to talk to the class, listen to their classmates, and critically analyze their own work and the work of their peers (pages 140, 190, 208). Developing communication skills will be a year-long goal for these students, a goal they will carry on with them indefinitely

Grade 2- Teacher Edition: 2, 4, 9, 76-77, 197-198, 228-229, 376, 382

Second grade students continue to learn to talk about math in their everyday lives and engage in acting out real-life problem situations (pages 2, 76-77). Math Talk can clarify student's thinking and expose the entire class to other approaches (pages 197-198, 376, 382). The discussion on pages 197-198 allows students to see that there are a variety of ways to solve a change plus word problem. This illustrates to the class that whether you use an equation, a circle drawing, or a math mountain to solve the problem, the solution of 7 computers is always the same. Students describe, explain, discuss and defend their solution methods when problem solving. They learn to talk to the class, listen to their classmates, and critically analyze their own work and the work of their peers (pages 4, 9, 228-229, 376, 382). Developing communication skills will be a year-long goal for these students, a goal they will carry on with them indefinitely

See also the Houghton Mifflin Harcourt Math Expressions © 2009 correlations [Grades K, 1, 2] to the Common Core State Standards Initiative for Mathematics (2010) for additional opportunities to see the integration of the math practices.

Mathematical Practice 4: Model with mathematics.

Mathematical Practice 4 states that Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation.

Math Expressions takes a modeling approach throughout in which students are provided with ample opportunities to use their prior mathematical knowledge to solve problems and represent them with models. Students model in their own ways with math drawings that show the problem situation or the quantities (including multi-digit quantities) involved. Much of the description of Mathematical Practices 1, 2, and 3 above involves students modeling. In K-2, problems arising in everyday life are more accessible to students than are those from society or the workplace, so K-2 concentrates on such problems.

Prior Knowledge
Students can apply the mathematics they know to help them solve problems arising in everyday life. *Daily Routines* and the *Quick Practice* routines (located in the Keeping Skills Sharp section of each lesson in the Teacher's Edition) help students master basic concepts, review previously learned content, and occasionally prepare the ground for new concepts before they are introduced. In the Teacher Edition, there are also *Ask for Ideas* headlines within the teaching activities that provide teachers with suggestions on how to activate students' prior knowledge and link previously learned content to the new information.

Models
Using prior knowledge is not the only way students solve problems. They are also able to identify important quantities in a practical situation and then model their relationships using a variety of mathematical models. Older students may use tools such as diagrams, tables, graphs, and formulas to model situations, but students at this younger grade span use the following models to represent math situations:

Scenarios- The main purpose of scenarios is to demonstrate mathematical relationships in a visual and memorable way. In scenario-based activities, a group of students is called to the front of the classroom to act out a particular situation. Scenarios are a useful model when a new concept is being introduced for the first time and they are especially valuable for demonstrating the physical reality that underlies such math concepts of embedded numbers and regrouping.

Manipulatives - Unique conceptual support materials and standard commercial manipulatives are used throughout the program. The unique, specially designed *Math Expressions* manipulatives (i.e. MathBoards, Secret Code Cards and Count On Cards) were developed during years of classroom research and they support students with their conceptual understanding as well as help build fluency.

Visual representations/Math Drawings- Initially, special manipulatives are used within the program, but the program rapidly moves to the use of math drawings (with the exception of Kindergarten) to represent mathematical situations. Math drawings support Math Talk because they can be done on the board for everyone to see during explanations of math thinking. They leave a record of students' thinking during class work and homework so teachers can get a feel for student approaches and identify errors. They eliminate logistical and cost issues that arise with manipulatives and provide continual experience with two-dimensional spatial thinking. They allow students to take pride in drawings they produce and they help reduce attention-consuming issues (fiddling, dropping things, off-task work, etc.). Math drawings focus on core math ideas and structures, and they provide experiences with fundamental math notations and concepts. They also make it easier to link the meanings in the drawing to numerical problems and computations. Such links can be made with circles, arrows, or other symbols.

Equations- Although students continue to use visual representations, they begin to accompany their drawings with symbolic representations in the form of expressions and equations. Using these expressions and equations help students make sense of quantities and their relationships in problem situations.

Supporting citations for Mathematical Practice 4:

It is clear from the following citations that children:

- apply what they know.
- are comfortable making assumptions and approximations to simplify a complicated situation.
- are able to identify important quantities in a practical situation.
- Model mathematics using tools (i.e. equations, manipulatives, drawings, diagrams, tables, graphs, and formulas).

Kindergarten- Teacher Edition: 9, 41, 139, 145, 360, 396, 408, 428, 436, 464, 617

To solve problems and conceptualize mathematics concepts, students use their prior mathematical knowledge. An example of this can be seen on page 617. Throughout the early units in the program, students had many opportunities to explore the partners of numbers. For example they saw that 7 could be broken into 1 and 6, 2, and 5, 3 and 4, and so on. In this activity, where students have to make equal groups, they can rely on their knowledge of partners to help them solve the problems.

They also represent mathematical situations with models, which include:

Using a scenario to act out and model a situation as seen on pages 139, 145, and 436.

Using manipulatives as seen on page 9 in which students use square tiles to model the embeddedness of numbers. Through the use of models, they see that 3 is made up of 2 partners (addends) and 1 partner, and 4 is made up of 2 partners and 2, or 3 partners and 1. The concept of partners and their total builds the foundation for addition. On page 41, the square inch tiles provide a model for students to represent comparison situations and conceptualize *more, fewer*, etc. On page 396, students model teen numbers with base ten cubes to see the ten and extra ones that compose a teen number like 15 (10 and 5).

Using mathematical drawings as seen on page 360. At this stage of mathematical development the teacher makes a math drawing to represent what students have modeled with their tiles. On pages 408 and 617, students are transitioning to making their own drawings and are shown how a matching drawing can represent a comparison situation.

Using symbolic notation, or expressions as seen on page 360. At this stage of mathematical development the teacher writes an expression to represent what students have modeled with their tiles. On page 464, you can see students have transitioned to writing expressions themselves when explaining their work to their class.

Grade 1- Teacher Edition: 28, 40-42, 46, 64, 116, 122, 134-135, 178-179, 218-219, 262, 338, 721, 755

To solve problems and conceptualize mathematics concepts, students use their prior mathematical knowledge. An example of this can be seen on page 262. Throughout the early units in the program, students had many opportunities to explore the partners of numbers. For example they saw that 10 could be broken into 1 and 9, 2, and 8, 3 and 7, and so on. In this ,activity where students have to find out whether an answer will be more or less, they can rely on their knowledge of partners to help them solve the problems. Also, on pages 754-755, students use their knowledge of adding two-digit numbers and apply the same concepts to adding money amounts.

They also represent mathematical situations with models, which include:

Using a scenario to act out and model a situation as seen on pages 40-42, 64, and 116.

Using manipulatives as seen on page 28-29 in which students use counters and break-apart sticks to model the composition of numbers. Through the use of models, they see that 4 is made up of 2 partners and 2 or 3 partners and 1. The concept of partners and their total builds the foundation of addition. On page 46, students use stair steps to explore and represent *1-more* and *1-less patterns*. On page 338-339, students use MathBoards and Secret Code Cards to model a two digit number ($43 = 40 + 3$), which emphasizes our base 10 number system.

Using mathematical drawings as seen on page 28. Students look at circle drawings to model the embededness of numbers. Students see the various ways in which the number 4 can be made. On pages 116-117 and 122 students use a math drawing to represent a real-world problem situation. On page 218, students use a math mountain drawing as a way of representing a mathematical situation. As students' computational skills become more sophisticated, they begin to use drawings (page 721) to help them add multi-digit numbers.

Using symbolic notation, expressions and equations as seen on page 28. Students write expressions to represent how the number 4 can be broken up ($1 + 3$, $2 + 2$, $3 + 3$). On pages 134-135 and 178-179, when students solve a word problem they accompany their math drawings with the corresponding expression or equation.

Grade 2- Teacher Edition: 16-17, 68-69, 76-77, 84-85, 92, 158, 197, 321, 388, 620, 642, 644, 671, 847, 955, 981, 992, 1012-1013; Student Activity Book: 163, 169, 285, 417, 433, 447

To solve problems and conceptualize mathematics concepts, students use their prior mathematical knowledge. On pages 670-671, students use their knowledge of subtracting three-digit numbers and apply the same concepts to making change with money amounts.

They also represent mathematical situations with models, which include:

Using a scenario to act out and model a situation as seen on pages 76-77.

Using manipulatives as seen on page 68 in which students use Secret Code Cards to help them use the Counting On strategy to solve the word problem. In this example, to show that we already knew 5 inches of snow, we show the front numeral side of the 5 card and then to find out how many more inches we need to get 8 inches of snow, we need to count 3 dots (the back side of the 3 card). On page 84, students use coin strips and on page 92 they use Green Make-a-Ten Cards to model the Make a Ten strategy, both of which help them add teen numbers. On page 321, students use MathBoards and Secret Code Cards to model a three digit number (166 = 100 + 60 + 6), which emphasizes our base 10 number system. On pages 1012-1013, students use fraction strips to help them visualize and model a fractional situation.

Using mathematical drawings as seen on pages 16-17, 68, and 197 to represent a problem solving situation. On page 158, under the representation heading, students use math drawings to find as many possible ways to make 12 cents. As students' computational skills become more sophisticated, they begin to use drawings (pages 388, 620, 642, 671, 847) to help them add and subtract multi-digit numbers. Students even use multiplication arrays on pages 980-981 to model multiplication situations pictorially.

Using symbolic notation, expressions, and equations as seen on page 68-69 and 197. When students solve a word problem they accompany their math drawings with the corresponding equation. As students learn more mathematics, the ways they model mathematical situations become more varied and efficient as seen when students begin to learn multiplication. On pages 954-955, you can see that students make the connection between counting by threes (3+3+3+3=12) and then multiplying four groups of three (4 x 3 = 12).

See also the Houghton Mifflin Harcourt Math Expressions © 2009 correlations [Grades K, 1, 2] to the Common Core State Standards Initiative for Mathematics (2010) for additional opportunities to see the integration of the math practices.

Mathematical Practice 5: Use appropriate tools strategically.

Mathematical Practice 5 begins by stating: Mathematically proficient students consider the available tools when solving a mathematical problem. In the only explicit statement about tools applicable to the content standards for K through 2, Mathematical Practice 5 states: These tools might include pencil and paper, concrete models, a ruler. Throughout *Math Expressions*, students are exposed to a variety of tools and learn to use them to solve problems.

At first glance, one might think that this practice refers to technological or measurement tools exclusively. However, students are exposed to many other mathematical tools. As students see and use a variety of tools, they begin to realize the strengths and limitations of each one and gain the experience to determine when and how to use them when solving problems.

Technology Tools- The materials in *Math Expressions* are presented in a wide variety of stimulating formats and students are provided with many opportunities for using various multimedia tools within the Go Digital Classroom. For example, students can use *Destination Math®,* a research-based software program that provides effective instruction, intervention, tutorials, and problem-solving support. Through the use of the internet, students can use *Education Place®* (www.eduplace.com), a free and interactive website, where materials, eGlossaries, and Brain Teasers, can be accessed. There is also an *electronic Student Activity Book* that is eReader accessible and Section 508 compliant. Other programs like *Soar to Success Math®* uses activity masters and online activities that provide individualized practice to bring students to proficiency, and *Mega Math®* helps students practice skills and apply concepts through exciting math adventures.

Calculators- The *Math Expressions* program understands that calculator use can become important in middle school, where computations can get in the way of problem solving. In the Kindergarten through Grade 5 programs, however, problem solving and computation are continually interrelated so that computations are meaningful and become fluent. Also, as an equity concern, many districts do not have classroom sets of calculators in elementary schools. Because of this, the program was developed in a way that all main lessons and activities are accessible to all students that may not have calculators. To address the need for some state's calculator practice, however, there are connection activities throughout the program that give students the opportunity to explore how to use a calculator and compare the strengths and limitations of that tool.

Measurement Tools- Measurement concepts are an important part of the mathematics curriculum from pre-kindergarten through high school because of the practicality and pervasiveness of the topics in every-day life. Math Expressions highlights this by exposing students to many tools (i.e. rulers, balance scales, thermometers, clocks) and has students link areas of measurement to other areas of mathematics, for example, number and operations. Not only do students experience these tools, but they also learn to select appropriate tools and see the advantages and limitations that each tool has.

Manipulatives- Unique conceptual support materials and standard commercial manipulatives are used throughout the program. The unique, specially designed *Math Expressions* manipulatives (i.e. MathBoards, Secret Code Cards and Count On Cards) were developed during years of classroom research and these concrete models support students with their conceptual understanding as well as help build fluency.

Visual Representations- Initially, special manipulatives are used within the program, but the program rapidly moves to the use of math drawings (with the exception of Kindergarten) to represent mathematical situations. Math drawings support Math Talk because they can be done on the board for everyone to see during explanations of math thinking. They leave a record of students' thinking during class work and homework so teachers can get a feel for student approaches and identify errors. They eliminate logistical and cost issues that arise with manipulatives and provide continual experience with two-dimensional spatial thinking. They allow students to take pride in drawings they produce and they help reduce attention-consuming issues (fiddling, dropping things, off-task work, etc.). Math drawings focus on core math ideas and structures, and they provide experiences with fundamental math notations and concepts. They also make it easier to link the meanings in the drawing to numerical problems and computations. Such links can be made with circles, arrows, or other symbols. Math drawings are made on paper to support individual thinking and on the class board to support discussion.

Paper and Pencil- As students become more sophisticated with computation, they begin to add and subtract larger numbers. Although they begin grouping ones into tens and tens into hundreds with Secret Code Cards and Place Value drawings solely, they then transition to adding and subtracting using accessible algorithms (New Groups Below, Show All Totals, etc.). Once students have seen mathematically general methods to add and subtract, they are still encouraged to link computational methods with visual representations to help check their work for accuracy and help explain their solution method.

Supporting citations for Mathematical Practice 5:

It is clear from the following citations that children:

- should be familiar with tools appropriate for their grade and make sound decisions about when each of these tools might be helpful.
- consider the available tools (i.e. manipulatives (concrete models), paper and pencil computation, visual representations, rulers, calculators etc.) when solving a mathematical problem.

Kindergarten-Teacher Edition: Software support in every lesson (Destination Math, MegaMath, Soar to Success); 8-9, 119, 144, 186-187, 200, 202, 206, 267, 272, 283, 396, 409, 428, 664-666, 670, 674-677, 680, 697, 698

Kindergarten students are exposed to many mathematical tools and use them to solve problems. They can use technology tools such as software, Internet resources, and Electronic Student Activity Books. They have opportunities to use calculators as seen on pages 200 and 206 and they use measurement tools such as pan balance scales, clocks, calendars, and containers (pages 283, 664-666, 674-677, 696, 698). Students also use a variety of manipulatives, such as square inch tiles, number tiles, coin strips, a Number Pattern Poster, base ten blocks (as seen on pages 8, 9, 144, 202, 266, 267, 272, 396) to make the mathematics concepts concrete and tangible. Visual representations, such as a number line, matching circle drawings, math mountains (pages 119, 409, 428) also provide experiences with fundamental math notations and concepts. As students see and use these tools, they begin to realize the strengths and limitations of each one and gain the experience to determine when and how to use them when solving problems as seen on page 680.

Grade 1- Teacher Edition: Software support in every lesson (Destination Math, MegaMath, Soar to Success); 46, 141, 147, 166, 296, 338, 350, 552-553, 662, 684, 721, 866-867, 923

First Grade students are exposed to many mathematical tools and use them to solve problems. They can use technology tools such as software, internet resources, and Electronic Student Activity Books. They have opportunities to use calculators as seen on pages 296 and they use measurement tools such as stair steps, clocks, calendars, and inch rulers (pages 552-553, 662, 684). Students also use a variety of manipulatives, such as stair steps, number cards, Red Count On Cards, MathBoards, and Secret Code Cards (as seen on pages 46, 141, 166, 338, 350) to make the mathematics concepts concrete and tangible. Visual representations, such as place value drawings and a number line (pages 350, 923) also provide experiences with fundamental math notations and concepts. Finally, students use accessible algorithms and paper and pencil methods like the New Groups Below method to solve problems as seen on page 721. As students see and use these tools, they begin to realize the strengths and limitations of each one and gain the experience to determine when and how to use them when solving problems as seen on pages 552, 866, and 867.

Grade 2- Teacher Edition: Software support in every lesson (Destination Math, MegaMath, Soar to Success); 94, 112-113, 118-119, 167, 197, 346, 348, 357, 449, 646, 656, 722, 733, 741, 847, 1013, 1042-1043, 1056-1058, 1076

Second grade students are exposed to many mathematical tools and use them to solve problems. They can use technology tools such as software, internet resources, and Electronic Student Activity Books. They have opportunities to use calculators as seen on pages 94, 348, 656 and 1076 and they use measurement tools such as centimeter rulers, clocks, balance scales, and containers (pages 166-167, 449). Students also use a variety of manipulatives, such as Math Mountain cards, counters, polygons, transparent mirrors, connecting cubes, and fraction strips (pages 112-113, 357, 722, 733, 740, 1013, 1056) to make the mathematics concepts concrete and tangible. Visual representations, such as a number path, a number line, math mountains, and circle drawings, (pages118-119, 197, 346, 646) also provide experiences with fundamental math notations and concepts. Finally, students use accessible algorithms and paper and pencil methods like the Ungroup First and Expanded methods to solve problems as seen on page 847. As students see and use these tools, they begin to realize the strengths and limitations of each one and gain the experience to determine when and how to use them when solving problems as seen on pages 166-167, 348, 656, 1042-1043, 1056.

See also the Houghton Mifflin Harcourt Math Expressions © 2009 correlations [Grades K, 1, 2] to the Common Core State Standards Initiative for Mathematics (2010) for additional opportunities to see the integration of the math practices.

Mathematical Practice 6: Attend to precision.

Mathematical Practice 6 states: Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They calculate accurately and efficiently. In the elementary grades, students give careful and clear explanations to each other. A key built-in component of *Math Expressions* is the frequent exchange of problem-solving strategies, or Math Talk. Students are encouraged on a daily basis to use these Math Talk opportunities and Writing Prompts to communicate their mathematical thinking with others. They are encouraged to pay close attention to detail and use precise mathematical language when explaining their work. Students also learn to be precise with their work and understand that certain situations call for an *exact* degree of accuracy or precision, while other situations can be *estimated.*

Precise Mathematical Language

In *Math Expressions* the teacher approaches math from the student's points of view by eliciting their thinking and rich language use. The use of both common informal and formal mathematical language helps students develop the vocabulary needed to express their growing understanding, and also helps them link drawings to written math notation to facilitate math modeling. This process of using and validating students' own language and experiences while connecting them to standard language and symbols facilitates listening, speaking, writing, and helping competencies, in addition to improving math skill. Teachers are provided with support and materials that offer ongoing practice and vocabulary application, for example:

Language and Vocabulary Teaching Notes provide teachers with point-of-use support in the Teacher's Edition. These classroom tips provide teachers with ideas and/or explanations of the math vocabulary. *Math Expressions* Vocabulary terms for the unit are outlined and explained in the Math Background section of each unit's opening pages.

English Language Learners Notes provide teachers with helpful vocabulary questions, prompts, and/or activities to help students learn and use vital mathematical language.

Visual Representations of vocabulary words in the Student Activity Book glossary make vocabulary words come alive. Also, in the primary grades, the pictures and illustrations in the Student Activity Book help cue young learners when solving word problems.

Through many years of research, *Accessible Vocabulary Terms* were gathered and woven into the learning materials. All of these terms capitalized on using effective language and solution methods that students could relate to and use in the classroom.

Using *Math Talk* in the classroom allows students an opportunity to share their problem-solving strategies and solutions as well as apply math vocabulary in context. Specific classroom organizational structures and activities help students develop the skills and confidence necessary to explain and justify their solutions. Support in the

Math Expressions Teacher's Edition offers a wealth of learning activities that directly support language development, questioning procedures, and student-to-student talk.

Body and Space Activities- These Kindergarten and First Grade activities help students with spatial orientation of their bodies and help build a precise vocabulary that can be linked to mathematical concepts. For example, in the Kindergarten body and space activity in Unit 1 Lesson 14, students practice standing in line and hearing the words "behind, in front of, and next to." These terms will later link into time order words and ordinal numbers studied in later units.

Not only do students use precise mathematical language, they also are encouraged to be precise and neat with their work computations and mathematical drawings to ensure accuracy.

Degree of Precision and Accuracy

Another area of accuracy and precision highlighted in *Math Expressions* is the exploration of the degree of precision used in problem solving and measurement situations. In some situations, it is not necessary for students to find an exact answer. Instead, they use strategies to
make estimates and use words like *almost* or *about* to describe a number. By exposing students to a variety of situations, they will learn to decipher when to use a precise or an approximate answer in their solution.

Supporting citations for Mathematical Practice 6:

It is clear from the following citations that children:

- try to communicate precisely to others and in their own reasoning, using clear definitions and formulated explanations.
- state the meaning of the symbols they choose and specify the units of measure with quantities in a problem.
- calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context.

Kindergarten- Teacher Edition: 1K, 41, 47, 60-61, 67, 95, 184, 198, 255, 278, 283, 300, 323, 506, 624, 661; Student Activity Book: 62, 158, 195

Using Math Talk, students use age appropriate explanations and precise mathematical language and vocabulary to communicate to others as they describe the properties and use the definitions of triangles, rectangles, and squares on pages 60-61. In another example, students use precise mathematical language to describe graphing and comparative situations on pages 40-41 and 322-323. They explain the commutative property with grade-appropriate explanations on page 255 and use precise language to compare and contrast three-dimensional figures and other objects on pages 46-47. They describe addition and subtraction situations using the scenario structures on pages 184, 198, 278 and teachers use *Anno's Counting Book* on page 506 to help students conceptualize math in the real world and give them more opportunities to use math vocabulary. To help teachers prepare for mathematical vocabulary terms, each unit opener outlines Math Expressions vocabulary as seen on page 1K. Students also learn to be precise with their work as they are encouraged to connect the dots in numerical order as seen on page 95. Students also understand that certain situations call for an *exact* degree of accuracy and precision. For instance in Unit 6 Lesson 4, students are only required to tell time to the hour or half hour, so the degree of accuracy doesn't have to be to the nearest minute.

Grade 1- Teacher Edition: 1K, 3, 21, 30, 41, 129, 185, 196, 226, 324, 623, 861, 866, 871, 878, 907; Student Activity Book: 71, 86, 308, 386

Using Math Talk, students use age appropriate explanations and precise mathematical language and vocabulary to communicate to others as they describe properties and use the definitions of squares, rectangles, triangles, and symmetrical objects (20, 226, 623, 878). On page 41, students arrange themselves concretely in front of the class to comprehend, visualize, and practice using ordinal numbers. In another example, students use precise mathematical language to connect break-aparts and equations as seen on page 30 and they also describe addition and subtraction equations on pages 185, 196, 197, 324, 325. To help teachers prepare for mathematical vocabulary terms, each unit opener outlines Math Expressions vocabulary as seen on page 1K. Students also learn to be precise with their work as they are encouraged to use their explanations and drawings to check their work as seen on pages 129, 196-197. They also learn to understand that certain situations call for an *exact* degree of accuracy and precision, while other situations can be *estimated* as seen on pages 860-861, 866, 871 and 907.

Grade 2- Teacher Edition: 174-175, 288-289, 295, 300-301, 309L, 326-327, 375-376, 586, 644-645, 806, 818-820, 856, 945, 987; Student Activity Book: 127, 157, 187, 195, 257

Using Math Talk, students use age appropriate explanations and precise mathematical language and vocabulary to communicate to others as they describe the properties and use the definitions of squares, rectangles, triangles, parallelograms, quadrilaterals and three-dimensional figures (174-175, 288-289, 295, 300-301, 586, 944-945). In another example, students use precise mathematical language to describe the steps in addition and subtractions computation methods as seen on pages 375, 376, 644-645, 818-820, 856. To help teachers prepare for mathematical vocabulary terms, each unit opener outlines Math Expressions vocabulary as seen on page 309L. Students also learn to be precise with their work as they are encouraged to draw neat representations of their place value drawings as seen on page 327. After a student explains their thinking, other students routinely ask questions to help extend and clarify the original explanation. The teacher also helps students extend and clarify. These practices help everyone move toward increased precision in all topics.

See also the Houghton Mifflin Harcourt Math Expressions © 2009 correlations [Grades K, 1, 2] to the Common Core State Standards Initiative for Mathematics (2010) for additional opportunities to see the integration of the math practices.

Mathematical Practice 7: Look for and make use of structure.

Mathematical Practice 7 states: Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have.

When solving problems, *Math Expressions* encourages students to identify patterns and structure continually and to make generalizations. Students are continually identifying mathematical structure in quantities, spatial objects, measure situations, and in problem situations and representing and discussing such structure.

Prior Learning and Generalizations

To solve problems and conceptualize concepts, students use their prior mathematical knowledge. Throughout early units in the program, Kindergarten and First Grade students had many opportunities to explore the partners of numbers. In later activities, they can rely on that knowledge of partners to help them solve more complex problems involving addition and subtraction. For instance, they saw that 7 could be broken into 1 and 6, 2, and 5, 3 and 4, and so on. In a later activity, where students have to make equal groups, they can rely on their knowledge of partners to help them solve the problems. In another example, they saw that 10 could be broken into 1 and 9, 2, and 8, 3 and 7, and so on. In a later activity, students have to find out whether an answer will be *more* or *less*, and using their knowledge of partners will assist them with their solutions.

In Grades One and Two, students use their knowledge of adding two-digit numbers and subtracting three-digit numbers and generalize those principles and apply them to the concepts of buying and selling items involving money amounts. Later in the year, students who have made sense of the strategies based on properties for finding totals using count-bys will be more likely to apply those properties when exploring multiplication.

Patterns and Properties

At this grade span, young students explore the idea of commutativity to help them understand that $2 + 5$ is the same as $5 + 2$ for example. They do this in concrete, visual ways through acting out and modeling with counters. Kindergarten students do this on the Counting Mat with objects. They generalize the pattern for +1 and -1. First graders see and draw models for partner switches to see the addends exchange places. Second graders use a Partner House and expressions to illustrate the property as well. Students also have an informal introduction to the Identity Property of Multiplication in which they see that multiplying by 1 does not change the value of a number.

Students not only use numeric patterns, but they are exposed to many concrete patterns using actions, shapes, and objects as well.

Structure

It is the aim of *Math Expressions* for students to conceptualize the base ten number system deeply and flexibly and understand the structure of numbers. When students understand how numbers are composed and decomposed, they can add and subtract fluently and accurately. For example, students use the Make-a-Ten strategy to help them add numbers like 8 + 6 to change it to an easier problem 10 + 4. They know that 8 + 2 will make a 10. So, take 2 in the 6 and and 4 more are in the 6. So 10 + 4 = 14 or 8 + 6 = 14. Kindergarten and Grade 1 students build teen numbers with base ten blocks to see the ten and extra ones that compose a teen number like 15 (10 and 5). At a later time, they will be able to use this ten-based structure and apply it to a situation that has them adding numbers like 15 and 17. For instance, some students struggle with the typical addition computation method in which they would add 5 and 7 to make 12 and "carry" the number. In our program, students can use the structures of numbers and expand the numbers to help with their addition. For example:

$$\begin{array}{l} 15 = 10 + 5 \\ \underline{17 = 10 + 7} \\ \quad 20 + 12 = \\ \quad 20 + 10 + 2 = 32 \end{array}$$

In Grade Two for instance, the same applies, but the numbers are larger and the situations are more complex as seen in Unit 5 Lesson 10.

Students also have opportunities to see the structure of geometric shapes and use the structure to help them classify and sort objects by certain criteria.

Supporting citations for Mathematical Practice 7:

It is clear from the following citations that children:

- analyze problems to discern a pattern or structure.
- draw conclusions about the structure they have identified.

Kindergarten- Teacher Edition: 186-187, 202-203, 209, 222-223, 257, 267, 275, 414

When solving problems, students recall prior learning as seen on page 617. Throughout the early units in the program, students had many opportunities to explore the partners of numbers. For example they saw that 7 could be broken into 1 and 6, 2, and 5, 3 and 4, and so on. In this activity, where students have to make equal groups, they can rely on their knowledge of partners to help them solve the problems.
They also identify +1 and –1 patterns on the Number Pattern poster and the 1-20 Boards. On pages 186, 202-203, 222-223, students, with some teacher guidance, should see that as each number increases, the same repeated quantity of 1 is added to that number. 2 + 1 is 3 and 3 + 1 is 4 and so on. Students can generalize, after seeing repeated calculations, that any number plus 1 will be the next greater number and the same applies for any number less 1. On pages 257 and 275, they can also see a pattern that arises in relation to the commutative property. Students "switch the partners," or physically rearrange the order of 3 students and 1 student to 1 student and three

students (3 + 1 or 1 + 3), allowing them to make the generalization that it doesn't matter in which order addends are added- they still total the same sum (4). On pages 266-267 and 414, students investigate the structure of numbers as they use number tiles, coins, or base ten blocks to see how a teen number is composed of one ten and some extra ones (12 = 10 + 2). This helps students with the tenness of numbers and ultimately will help them with addition and subtraction later on.

Grade 1- Teacher Edition: 42-43, 64, 262, 324, 338-339, 461, 754-755

When solving problems, students recall prior learning as seen on page 262. Throughout the early units in the program, students had many opportunities to explore the partners of numbers. For example they saw that 10 could be broken into 1 and 9, 2, and 8, 3 and 7, and so on. In this activity where students have to find out whether an answer will be *more* or *less*, they can rely on their knowledge of partners to help them solve the problems. Also, on pages 754-755, students use their knowledge of adding two-digit numbers and apply the same concepts to adding money amounts. They also identify +1 and –1 patterns using Giant Number Cards and acting out the pattern, making the pattern tangible and concrete. On pages 42, students, with some teacher guidance, should see that as each number increases, the same repeated quantity of 1 is added to that number. 2 and 1 more is 3 and 3 and one more is 4 and so on. Students can generalize, after seeing repeated calculations, that any number plus 1 will be the next greater number and the same applies for any number less 1. On page 64, they can also see a pattern that arises in relation to commutativity. Students "switch the partners," or physically rearrange the order of 5 students and 2 students to 2 students and 5 students (5 + 2 or 2 + 5), allowing them to make the generalization that it doesn't matter in which order addends are added- they still total the same sum (7). On pages 324-325, students investigate the structure of numbers as they use a MathBoard and stair steps to see how a teen number is composed of one ten and some extra ones (9 + 4 = 10 + 3). This helps students with the tenness of numbers and ultimately will help them with addition and subtraction later on. They continue exploring the structure of two digit numbers as seen on page 338-339.

Grade 2- Teacher Edition: 14-1552, 84-85, 150-151, 300-301, 376-377, 586, 670-671, 682-683, 847, 954-955, 958, 975; Student Activity Book: 23, 67, 257, 299, 432

When solving problems, students recall prior learning as seen on page pages 670-671. Students use their knowledge of subtracting three-digit numbers and apply the same concepts to making change with money amounts. Also, on pages 954-955, you can see that students make the connection between skip counting by threes (3+3+3+3=12) and then applying that knowledge when multiplying four groups of three (4 x 3 = 12). They also identify Zero, and +1/–1 patterns using circle drawings, making this pattern tangible and concrete. On pages 14 and 15, students, with some teacher guidance, should see that the following rule can be generalized: adding or subtracting zero from a number does not change that number (5 + 0 = 0 and 7 – 0 = 0). They can also see that as each number increases or decreases, the same repeated quantity of 1 is added or subtracted to that number. 7+ 1 = 8 and 5 – 1 = 4 and so on. Students can generalize, after seeing repeated calculations, that any number plus 1 will be the next greater number and the same applies for any number less 1. On page 52, they can also see a pattern that arises in relation to commutativity. Students "switch the partners," or rearrange the order of partners/addends in the partner houses. They show that 1 + 9 and 9 + 1 always equal 10. On page 682, they use a math mountain and write 8 equations to show that 59 + 24 = 24 + 59, both examples allowing them to make the generalization that it doesn't matter in which order addends are added- they still total the same sum. On pages 324-325, students investigate the structure of numbers as they use coin strips and fingers to see how a teen number is composed of one ten and some extra ones (8+ 6 = 10 + 4 = 14). This helps students with the tenness of numbers and ultimately will help them with addition and subtraction later on. They continue exploring the structure of two and three digit numbers as seen on pages 376-377 and 847 and use this structure when adding and subtracting numbers with accessible algorithms. Students also explore the structure of equations with three addends on pages 150-151 and make generalizations about the Associative Property of Addition.

See also the Houghton Mifflin Harcourt Math Expressions © 2009 correlations [Grades K, 1, 2] to the Common Core State Standards Initiative for Mathematics.

Mathematical Practice 8: Look for and express regularity in repeated reasoning.

Mathematical Practice 8 states: Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts.

Math Expressions provides students with many opportunities to repeat calculations and solve similar problem types and situations. In doing this, they can devise mathematical short cuts to help them solve future situations in a mathematical efficient and effective manner. Students are supported to move through the learning progression of levels of addition and subtraction methods described in the OA standards for K through 2. This progression involves general methods that are more abstract and abbreviated; each level is a short-cut of the level before.

It is the aim of Math Expressions to give students the opportunities to regularly see that if calculations are repeated, they can look for general methods and find short cuts or strategies to help them solve problems.

1-More and 1-Less Patterns- Kindergarten students review, discuss, and find as many patterns as they can on the Number Pattern Poster. Students, with some teacher guidance, should see that as each number increases, the same repeated quantity of 1 is added to that number. 2 + 1 is 3 and 3 + 1 is 4 and so on. Students can generalize, after seeing repeated calculations, that any number plus 1 will be the next greater number and the same applies for any number less 1.

Count All vs. Count On- The Count All strategy is an addition strategy where students count all of something (for example, fingers, counters, circles, dots on Secret Code Cards) to find the total. If you wanted to add 5 and 4, you would count *all* the objects 1, 2, 3, 4, 5 … 6, 7, 8, 9. Alternatively, the Count On strategy is an addition or subtraction strategy in which students begin with one partner (addend) and count on to the total. If you wanted to add 5 and 4, you would start at "I already have 5… 6 7 8 9. The Count On strategy differs from the Count All strategy in that the counting is abbreviated by counting on from the greater number. This consolidated approach, or short cut, is especially important when students start adding numbers with totals greater than 10. These strategies are illustrated on pages 141 and 219 (Grade 1).

Adding Up- This strategy is a way for students to progress from the counting on method of addition and apply it to the subtraction of multi-digit numbers. In the second grade example (Unit 9 Lesson 14) of 151 – 84, it is not efficient for students to count backward 84 from 151. Students can use the Adding Up short cut instead.

Start at 4 and add to the next 10.	84 + 6 = 90
Then we count by tens to 150:	= 100, 110, 120, 130, 140, 150 (6 tens)
Then we count one more to 151:	+1 = 151
That's 6 tens and 7 ones, or 67.	84 + 67 = 151

Someone else may see a short cut and solve it this way:

I'd add 6 to 84 to get 90.	84 + 6 = 90
Then I'd add 10 to 90 to get to 100.	+10 = 100
Then I'd add 51 to get to 151.	+51 = 151

Then add 51 + 10 + 6 = 67 84 + 67 = 151

The continual Math Talk about student's ways of representing and solving problems shares with the whole class more efficient short-cuts certain students develop.

Multiplication- As students learn more mathematics, the ways they model mathematical situations become more varied and efficient as seen when students begin to learn multiplication. On pages 954-955 (Grade 2), you can see that students make the connection between counting by threes (3+3+3+3=12) and then multiplying four groups of three (4 x 3 = 12). Students begin to see they can use multiplying as a shorter, more efficient way of adding equal groups.

Division- As students learn more mathematics, the ways they model mathematical situations become more varied and efficient as seen when students begin to learn division. On pages 994 (Grade 2), you can see that students make the connection between repeated subtraction (15-3 = 12- 3 = 9-3 = 6-3 = 3-3 = 0) and then dividing 15 by 3 equal groups (15 ÷ 3= 5). Students begin to see they can use division as a shorter, more efficient way of dividing equal groups.

Properties- Second grade students (page 958) also have an informal introduction to the Identity Property of Multiplication in which they see that multiplying by 1 does not change the value of a number. Students can generalize, after seeing repeated calculations that any number multiplied by 1 will not change the value of a number.

Supporting citations for Mathematical Practice 8:

It is clear from the following citations that children:

- look for and express regularity in repeated reasoning.
- notice if calculations are repeated and look both for generalizing methods and for shortcuts.
- maintain oversight of the process, while attending to the details.
- continually evaluate the reasonableness of their intermediate results.

Kindergarten- Teacher Edition: 186, 203, 223, 327, 463

Students have many opportunities to regularly see that if calculations are repeated, they can look for general methods and find short cuts or strategies to help them solve problems. They identify +1 and –1 patterns on the Number Pattern poster and the 1-20 Boards. On pages 186, 202-203, 222-223, 327, and 463, students, with some teacher guidance, they should see that as each number increases, the same repeated quantity of 1 is added to that number. 2 + 1 is 3 and 3 + 1 is 4 and so on. Students can generalize, after seeing repeated calculations, that any number plus 1 will be the next greater number and the same applies for any number less 1.

Grade 1- Teacher Edition: 42, 46, 141, 219, 947

Students have many opportunities to regularly see that if calculations are repeated, they can look for general methods and find short cuts or strategies to help them solve problems. Students, with some teacher guidance, identify +1 and –1 patterns using

Giant Number Cards and acting out the pattern, making this pattern tangible and concrete. On pages 42 and 46, students should see that as each number increases, the same repeated quantity of 1 is added to that number. 2 and 1 more is 3 and 3 and one more is 4 and so on. Students can generalize, after seeing repeated calculations, that any number plus 1 will be the next greater number and the same applies for any number less 1. On pages 140-141 and 219, students learn the advantages of the Counting On strategy over the Counting All strategy. This consolidated approach, or short cut, is especially important when students start adding numbers with totals greater than 10. Students also use short cuts when multiplying as seen on page 947. Students will see that multiplying and with equal groups is quicker than using skip counting/count-bys.

Grade 2- Teacher Edition: 14-15, 60-61, 692, 955, 957, 958, 994

Students have many opportunities to regularly see that if calculations are repeated, they can look for general methods and find short cuts or strategies to help them solve problems. As seen on pages 14 and 15, students, with some teacher guidance, should see the following rule: adding or subtracting zero from a number does not change that number ($5 + 0 = 0$ and $7 - 0 = 0$). They can also see that as each number increases or decreases, the same repeated quantity of 1 is added or subtracted to that number. $7+1 = 8$ and $5 - 1 = 4$ and so on. Students can generalize, after seeing repeated calculations, that any number plus 1 will be the next greater number and the same applies for any number less 1. By the end of the grade, students progress from addition properties and begin to make generalizations about multiplication. On page 958, they have an informal introduction to the Identity Property of Multiplication in which they see that multiplying by 1 does not change the value of a number. Students can generalize, after seeing repeated calculations that any number multiplied by 1 will not change the value of a number. On pages 60-61, students learn the advantages of the Counting On strategy over the Counting All strategy. This consolidated approach, or short cut, is especially important when students start adding numbers with totals greater than 10. As students become more sophisticated with their computation, they can use the Adding Up short cut as seen on page 692. Students also use short cuts when multiplying and dividing as seen on pages 955 and 994. Students will see that multiplying and dividing with equal groups is quicker than using skip counting/count-bys or repeated subtraction.

See also the Houghton Mifflin Harcourt Math Expressions © 2009 correlations [Grades K, 1, 2] to the Common Core State Standards Initiative for Mathematics (2010) for additional opportunities to see the integration of the math practices.